BASIC CONCEPTS

1)

If a, b, c are distinct positive integers, and

$$\frac{a}{b}+1=c\ ,\ a+b=12$$

then what is the sum of all possible values for b?

A) 6 B) 10 C) 15 D) 18 E) 22

Solution:

Positive integers start at 1 and continue as 1, 2, 3, and so on.

If we make some adjustments to the first equation provided in the question, we find the following

$$\frac{a}{b}+1=c \Rightarrow \frac{a}{b}+\underset{(b)}{\frac{1}{1}}=c \Rightarrow \frac{a+b}{b}=c \Rightarrow a+b=bc$$

It is given in the question that a + b is equal to 12. Let's substitute this value into the expression we found

$a+b=bc \Rightarrow bc=12$

Let's find the values of b and c by substituting numbers.

For $b=1$, $c=12$, $a=11$

For $b=2$, $c=6$, $a=10$

For $b=3$, $c=4$, $a=9$

For $b=4$, $c=3$, $a=8$

For $b=6$, $c=2$, $a=6$ a, b, and c must be distinct from each other. We cannot use them.

For $b=12$, $c=1$, $a=0$ a is not a positive integer. We cannot use them.

The sum of the values taken by b is $1+2+3+4=10$

Correct Answer: B

2)

Given that a, b, and c are positive integers and

$a\cdot b=5$

$a\cdot c=15$

What is the smallest possible value of $a+b+c$?

A) 9 B) 10 C) 11 D) 12 E) 13

Solution:

In these types of questions, the variable that appears in multiple products should be assigned the largest possible value. Therefore, the variable "a" should be set to 5.

If $a.b=5$ then $b=1$

If $a.c=15$ then $c=3$

We obtained $a+b+c=5+1+3=9$

Correct Answer: A

3)

a, b, and c are positive integers and

$a\cdot b=12$

$a\cdot c=16$

What is the largest possible value of $a+b+c$?

A) 11 B) 21 C) 29 D) 32 E) 33

Solution:

In this question,similar to the one above, the largest total value is sought. To achieve the maximum total value, the common factor should be as small as possible. Therefore, the variable "b" is assigned the smallest positive integer, which is 1.

If $a.b=12$, then $a=12$ is obtained.

If $b.c=16$, then $c = 16$ is obtained.

$a+b+c=12+1+16$ results in 29.

Correct Answer: C

4)

a, b, and c are integers and

$a\cdot b=12$

$a\cdot c=16$

What is the smallest possible value of $a+b+c$?

A) -10 B) -11 C) -20 D) -24 E) -29

Solution:

In this case, since the numbers are integers, you have the option to use negative integers as well.

To minimize the total value of the numbers, it's best to use negative integers.

When working with negative integers, you should think in the opposite way compared to operations with positive integers.

To achieve the smallest total value, the common factor should be the largest negative integer.

So, in this case, the variable "b" is set to the largest negative integer, which is -1 (not -4).

If $a.b = 12$, then $a = -12$

If $b.c = 16$, then $c = -16$

$a + b + c = (-12) + (-1) + (-16) = -29$ is the result.

Correct Answer: E

5)

Given that a and b are distinct positive integers, and

$a + b = 12$

What is the sum of the minimum and maximum possible values for the expression $a \cdot b$?

A) 35 B) 40 C) 42 D) 46 E) 47

Solution:

In situations where the sum is given and you want to maximize the product, the numbers should be chosen close to each other. Conversely, when you want to minimize the product, the numbers should be selected far from each other.

In the question, $a + b = 12$ is given

The closest numbers $\Rightarrow a = 6,\ b = 6$ (we cannot choose the same numbers since they need to be distinct)

The closest numbers $\Rightarrow a = 7,\ b = 5 \Rightarrow a.b = 35$ (The largest value)

The closest numbers $\Rightarrow a = 1,\ b = 11 \Rightarrow a.b = 11$ (The smallest value)

The sum of 35 and 11 is 46.

Correct Answer: D

6)

a and b are negative integers, and

$a + b = -10$

What is the minimum value of the product $a \cdot b$?

A) 9 B) 16 C) 24 D) 25 E) 27

Solution:

To minimize their product, the numbers need to be chosen far apart from each other.

In the given equation $a + b = -10$,

the farthest numbers are $a = -1$ and $b = -9$

The product $a.b$ equals 9, which is the smallest possible value.

Correct Answer: A

7)

a and b are integers, and

$a \cdot b = 15$

How much greater is the largest value of $a \cdot b$ compared to the smallest value of $a \cdot b$?

A) 0 B) 8 C) 20 D) 24 E) 32

Solution:

<u>To obtain the largest value of $a+b$</u>

a and b should be positive, and they should be far apart from each other.

In this case, if $a=15$ and $b=1$, then $a+b=16$

<u>To obtain the smallest value of $a+b$</u>

a and b should be negative, and they should be far apart from each other.

In this case, if $a=-15$ and $b=-1$, then $a+b=-16$

The difference is $16-(-16)=32$

Correct Answer: E

8)

a and b are natural numbers, and

$$a \cdot b = 8$$

What is the smallest value of the expression $5a - 2b$?

A) -16 B) -13 C) -11 D) 0 E) 12

Solution:

To find the smallest value of the expression $5a-2b$,
b should be the largest, and a should be the smallest.
For the equation $a.b = 8$, the smallest a is 1,
and the largest b is 8.
Therefore, the smallest value of $5a-2b$ is
$5.1-2.8 = 5-16 = -11$

Correct Answer: C

9)

$$\frac{x}{3^2.5}+\frac{y}{2^2.3}-\frac{z}{2.3.5}=\frac{1}{18}$$

What is the value of the expression $4x + 15y - 6z$?

A) 8 B) 10 C) 12 D) 14 E) 16

Solution:

Let's equalize the denominators in the expression

$$\frac{x}{3^2.5}+\frac{y}{2^2.3}-\frac{z}{2.3.5}=\frac{1}{18}$$

To make all the denominators have the same factors, we can expand them as follows:

$$\underset{(2^2)}{\frac{x}{3^2.5}}+\underset{(3.5)}{\frac{y}{2^2.3}}-\underset{(2.3)}{\frac{z}{2.3.5}}=\frac{1}{18} \Rightarrow \frac{4x}{2^2.3^2.5}+\frac{15y}{2^2.3^2.5}-\frac{6z}{2^2.3^2.5}$$

$$\Rightarrow \frac{4x+15y-6z}{2^2.3^2.5}=\frac{1}{18}$$

$$\Rightarrow \frac{4x+15y-6z}{180}=\frac{1}{18} \Rightarrow 4x+15y-6z=\frac{180}{18}$$

$$\Rightarrow 4x+15y-6z=10 \text{ is obtained.}$$

Correct Answer: B

10)

a, b, c are distinct integers, and

$$a \cdot b = \frac{24}{c}$$

What is the minimum possible value of the sum $a+b+c$?

A) -24 B) -9 C) 0 D) 9 E) 26

Solution:

In the equation $a.b=\frac{24}{c}$, if we move 'c' to the other side, we get $a.b.c=24$

To minimize their sum, two of the numbers should be negative, and one should be positive.
By choosing 'a' as the smallest positive integer, which is 1, we ensure that the other two negative numbers are as far apart as possible.

So, we set $a=1$, $b=-24$, and $c=-1$
Then, $a+b+c=1+(-24)+(-1)=-24$
This way, we achieve the smallest sum

Correct Answer: A

11)
a, b, and c are single digits.
If $a = 2b$ and $2a = c$,
What is the sum of the <u>minimum</u> and <u>maximum</u> possible values of $a + b + c$?

A) 7 B) 10 C) 12 D) 14 E) 21

Solution:
The digits are 0, 1, 2, 3, ..., 9.
The question states that $a = 2b$ and $2a = c$
By substituting 2b for a in the equation $2a = c$,
we obtain the equation $4b = c$
The values of a, b, and c are then 2b, b, and 4b, respectively.
To find the largest values, we should assign the highest possible value to b. However, the expression 4b should not exceed the highest digit, which is 9.
Therefore, b can be at most 2.
The largest values for a, b, $c \Rightarrow 2b$, b, $4b \Rightarrow 4$, 2, 8
which results in $a + b + c = 14$
For the smallest value, we can choose b to be 0.
Since the question did not specify that the digits should be distinct, there is no issue with the digits being the same.
The values for a, b, and c are then 2b, b, and 4b
0, 0, 0, which results in $a + b + c = 0$
So, the total is $14 + 0 = 14$.

Correct Answer: D

12)
a, b, and c are negative integers.

$$\frac{a}{b} = \frac{3}{4} \text{ ve } c = 3b$$

What is the maximum possible value of the sum $a + b + c$?

A) 6 B) 11 C) -5 D) -17 E) -19

Solution:
First, it's helpful to express the equation $c = 3b$ in fractional form.

$$c = 3b \Rightarrow \frac{c}{b} = \frac{3}{1}$$

We now have two fractional equations. In such questions, it's convenient to make the common factor of the coefficients the same to reach a solution more easily.

$$\frac{a}{b} = \frac{3}{4} \text{ ve } \frac{c}{b} = \frac{3}{\underset{(4)}{1}} \Rightarrow \frac{a}{b} = \frac{3}{4} \text{ ve } \frac{c}{b} = \frac{12}{4}$$

By making the coefficients of the common factor the same, we can express a, b, and c as multiples of a common factor, let's call it "k."
This results in $a = 3k$, $b = 4k$, and $c = 12k$.
Since a, b, and c are stated to be negative, we choose k to be -1 to maximize their values.
As a result, $a = -3$, $b = -4$, and $c = -12$

$$\Rightarrow a + b + c = (-3) + (-4) + (-12) = -19$$

Correct Answer: E

13)
a and b are natural numbers.
Given that $3a + 4b = 48$,
how many distinct values can $a + b$ take?

A) 5 B) 6 C) 7 D) 8 E) 9

Solution:
In the expression $3a + 4b = 48$, the possible values for a are multiples of the number in front of b, while the values for b are multiples of the number in front of a.
When a increases by 4 units, b decreases by 3 units.
When $a = 0$, $b = 12$, $a + b = 12$
When $a = 4$, $b = 9$, $a + b = 13$
When $a = 8$, $b = 6$, $a + b = 14$
When $a = 12$, $b = 3$, $a + b = 15$
When $a = 16$, $b = 0$, $a + b = 16$

So, $a + b$ takes 5 different values.

Correct Answer: A

14)

a, b, and c are distinct natural numbers.

$$5a+4b+3c=75$$

What is the largest possible value that c can take?

A) 19 B) 20 C) 21 D) 22 E) 23

Solution:

In the expression $5a+4b+3c=75$, to find the largest possible value for c, we start by assigning the lowest possible values to the coefficients other than c.

We can use the smallest natural number, which is 0, for a. Since the question states that the numbers are distinct, we assign the next smallest natural number, 1, to b

$5a+4b+3c=75$

$\downarrow \quad \downarrow \quad \downarrow$

$0 \quad 1 \quad x \quad \Rightarrow$ c does not come out as an integer.

$0 \quad 2 \quad x \quad \Rightarrow$ c does not come out as an integer.

$0 \quad 3 \quad 21 \quad \Rightarrow$ The value of c is found to be 21

Correct Answer: C

15)

Let x and y be two-digit natural numbers, and

$$x-y = 36$$

How many different values of x satisfy this equation?

A) 53 B) 54 C) 55 D) 56 E) 57

Solution:

In the expression $x-y=36$, to find how many different values x can take, we need to determine the largest and smallest values for x.

x can be as large as the largest two-digit natural number, which is 99

For the smallest value of x, we assign the smallest two-digit natural number, which is 10, to y

$\Rightarrow x-10=36 \Rightarrow x=46$ is the smallest value.

This means that x can take all values from 46 to 99.

No. of consecutive terms = Last Term − First Term + 1

The number of values for $x = 99-46+1=54$

Correct Answer: B