## ODD EVEN NUMBERS

## 1)

Which of the following is an even number?
A) $2^{0}+4^{3}$
B) $13^{2}+15^{2}-23^{2}$
C) $2^{5}-7^{2}-4^{3}$
$\begin{array}{ll}\text { D) } 7^{3}-4^{6}+5^{5} & \text { E) } 6^{5}+7^{5}\end{array}$

## Solution:

Let's examine the options one by one.
A) $2^{0}+4^{3}$
$2^{0} \rightarrow$ The zeroth power of any number is 1 . Therefore, it is odd.
$4^{3} \rightarrow$ All positive powers of even numbers are even.
$\Rightarrow 2^{0}+4^{3} \Rightarrow \mathrm{O}+\mathrm{E}=\mathrm{O}$
$\Rightarrow$ The number in option $A$ is odd.
B) $13^{2}+15^{2}-23^{2} \Rightarrow$ All natural number powers of odd numbers are odd.

$$
\Rightarrow 0+0-0=0
$$

$\Rightarrow$ The number in option $B$ is odd.
C) $2^{5}-7^{2}-4^{3} \Rightarrow \mathrm{E}-\mathrm{O}-\mathrm{E}=\mathrm{O}$
$\Rightarrow$ The number in option $C$ is odd.
D) $7^{3}-4^{6}+5^{5} \Rightarrow \mathrm{O}-\mathrm{E}+\mathrm{O}=\mathrm{E}$
$\Rightarrow$ The number in option $D$ is even.
E) $6^{5}+7^{5} \Rightarrow \mathrm{E}+\mathrm{O}=\mathrm{O}$
$\Rightarrow$ The number in option E is odd.
Correct Answer: D

## 2)

Assuming $a$ is an integer and that $7 a+4$ is an even number, which of the following is an odd number?
A) $a+4$
B) $5 a-2$
C) $a^{2}+a$
D) $a^{5}+2 \quad$ E) $a^{5}+4 a-3$

## Solution:

Since the expression $7 a+4$ is even, we can deduce inf ormation about the value of a.
$7 \mathrm{a}+4=\mathrm{E} \Rightarrow 7 \mathrm{a}+\mathrm{E}=\mathrm{E} \Rightarrow 7 \mathrm{a}=\mathrm{E} \Rightarrow \mathrm{a}$ is even. Let's examine the options one by one.
A) $a+4 \Rightarrow E+E=E$
B) $5 \mathrm{a}-2 \Rightarrow \mathrm{O} \cdot \mathrm{E}-\mathrm{E} \Rightarrow \mathrm{E}-\mathrm{E}=\mathrm{E}$
C) $a^{2}+a \Rightarrow E^{2}+E \Rightarrow E+E=E$
D) $a^{5}+2 \Rightarrow E^{5}+E \Rightarrow E+E=E$
E) $\mathrm{a}^{5}+4 \mathrm{a}-3 \Rightarrow \mathrm{E}^{5}+\mathrm{E} \cdot \mathrm{E}-\mathrm{O} \Rightarrow \mathrm{E}+\mathrm{E}-\mathrm{O}=\mathrm{O}$

Correct Answer: E

## 3)

Considering that $\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{m}$, and n are all positive int egers and that

$$
(a+b)^{c}=2 m+3 \text { ve }(b . c)^{a}=2 n
$$

which of the following is definitely true?
A) If $a$ is an even number, then $c$ is an even number.
B) If $b$ is an even number, then $c$ is an odd number.
C) $b$ is an even number.
D) $a$ is an odd number.
E) If $a$ is an odd number, then $c$ is an odd number.

## Solution:

The number " $2 m+3$ " is a single number, and " $2 n$ " is an even number. Therefore,
$(a+b)^{c}=0$ and $(b . c)^{a}=E$
$(a+b)^{c}=0 \Rightarrow a+b=0\left(0^{n}=0\right)$
$\Rightarrow$ One of the numbers $a$ and $b$ must be even, and the other must be odd.

$$
(\mathrm{b} . \mathrm{c})^{\mathrm{a}}=\mathrm{E} \Rightarrow \mathrm{~b} . \mathrm{c}=\mathrm{E} \quad\left(\mathrm{E}^{\mathrm{n}}=\mathrm{E}\right)
$$

$\Rightarrow$ At least one of the numbers $b$ and $c$ must be even.

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In this case,
If $a$ is even, then $b$ is odd, and $c$ is even. If $a$ is odd, then $b$ is even, and $c$ can be either odd or even

Only option A specifies a condition that meets these criteria; the other options do not provide definitive condition.

Correct Answer: A

## 4)

$a, b$, and $c$ are all int egers, and

$$
\frac{a}{12}=11 . b \cdot c
$$

Which of the following is definitely an even number?
A) $a c+b$
B) $a+2 b$
C) $a^{2}+b$
D) $2 \mathrm{c}-\mathrm{b}$
E) $a+b+c$

## Solution:

In the equation $\frac{a}{12}=11 . b . c$,
when we move 12 to the other side $\Rightarrow a=12.11$.b.c
Since 12 is even, a must be an even number.
We don't have a definite piece of inf ormation about b and c. Let's examine the options:
A) $\mathrm{ac}+\mathrm{b} \Rightarrow \mathrm{E} \cdot \mathrm{b}+\mathrm{b}=\mathrm{E}+\mathrm{b} \Rightarrow$ There is no certaint y
B) $a+2 b \Rightarrow E+E \cdot b \Rightarrow E+E=E \Rightarrow$ It is definitely even.
C) $a^{2}+b \Rightarrow E^{2}+b \Rightarrow E+b \Rightarrow$ There is no certainty
D) $2 c-b \Rightarrow E . c-b \Rightarrow E-b \Rightarrow$ There is no certainty
E) $a+b+c \Rightarrow E+b+c \Rightarrow$ There is no certainty

Correct Answer: B

## 5)

Given that $a, b$, and $c$ are even numbers, which of the following is always an even number?
A) $\frac{a+b-c}{2}$
B) $\frac{a+b+c}{2}$
C) $\frac{a+b}{2}+c$
D) $\frac{\text { a.b.c }}{2}$
E) $a+\frac{b-c}{2}$

## Solution:

Since $a, b$, and $c$ are even numbers, we can express them as follows: $a=2 x, b=2 y, c=2 z$
Now, let's examine the options:
A) $\frac{a+b-c}{2} \Rightarrow \frac{2 x+2 y-2 z}{2}=\frac{2(x+y-z)}{2}=x+y-z$
$\Rightarrow$ There is no certainty
B) $\frac{a+b+c}{2} \Rightarrow \frac{2 x+2 y+2 z}{2}=\frac{2(x+y+z)}{2}=x+y+z$
$\Rightarrow$ There is no certainty
C) $\frac{a+b}{2}+c \Rightarrow \frac{2 x+2 y}{2}+2 z=\frac{2(x+y)}{2}+2 z=x+y+E$
$\Rightarrow$ There is no certainty
D) $\frac{\text { a.b.c }}{2} \Rightarrow \frac{2 x .2 y .2 z}{2} \Rightarrow 4 x . y . z \Rightarrow E$
$\Rightarrow$ It is definitely even.
E) $a+\frac{b-c}{2} \Rightarrow 2 x+\frac{2 y-2 z}{2} \Rightarrow E+\frac{2(y-z)}{2} \Rightarrow E+y-z$
$\Rightarrow$ There is no certainty Correct Answer: D

