POSITIVE NEGATIVE NUMBERS

1)

Given the statements:

$a^3.b < 0$

$b^2.c > 0$

Which of the following is always true?

A) $a.b > 0$ B) $a.c > 0$ C) $b > 0$

D) $b.c < 0$ E) $a^2.c > 0$

Solution:

The product of numbers with the same sign is always positive, and the product of numbers with different signs is negative.

Even – degree expressions are always positive.

Odd – degree expressions take on the sign of the base.

$a^3.b < 0 \Rightarrow a.b < 0 \Rightarrow$ a and b have opposite signs

$b^2.c > 0 \Rightarrow (+).c > 0 \Rightarrow$ c is definitely positive.

We cannot say anything about b.

Now let's examine the answer choices:

A) $a.b > 0 \Rightarrow$ Since a and b have opposite signs, their product will always be negative.

B) $a.c < 0 \Rightarrow$ We know that c is positive, but we don't have information about the sign of a, so we cannot make a definite inference.

C) $b > 0 \Rightarrow$ We don't have enough information about the sign of b.

D) $b.c < 0 \Rightarrow$ We know that c is positive, but we don't have information about the sign of b, so we cannot make a definite inference.

E) $a^2.c > 0 \Rightarrow$ Since a has an even degree, it is always positive, and we know that c is positive. Therefore, the product $a^2.c$ is always positive.

Correct Answer: E

2)

Given that

$a.b^2 > 0$

$b^5.c > 0$

$a^3.c < 0$

What are the signs of a, b, and c, respectively?

A) $+,+,-$ B) $+,-,+$ C) $+,-,-$

D) $-,+,-$ E) $+,+,+$

Solution:

$a.b^2 > 0 \Rightarrow a.(+) > 0 \Rightarrow a \to +$ (b^2 is always positive.)

$b^5.c > 0 \Rightarrow b.c > 0 \Rightarrow$ b and c must have the same sign

$a^3.c < 0 \Rightarrow a.c < 0 \Rightarrow (+).c < 0 \Rightarrow c \to -$

In this case, the signs of a, b, and c should be $+,-,$ and $-$, respectively.

Correct Answer: C

3)

Given that,

$a < 0 < b < c$

which of the following is definitely positive?

A) $(a-b).(b-c)$ B) $(a+b).(b+c)$

C) $(a+c)(a-c)$ D) $(a-c).c$

E) $(a+c).b$

Solution:

If we examine the options one by one:

A) $\underset{(-)}{(a-b)}.\underset{(-)}{(b-c)} \Rightarrow (-).(-) \Rightarrow (+)$ When a larger number is subtracted from a number, the result is always negative

B) $\underset{(-)}{(a}+\underset{(+)}{b)}.\underset{(-)}{(b}+\underset{(+)}{c)} \Rightarrow$ Without knowing which of the absolute values of oppositely signed numbers is greater, we cannot determine whether the result is positive or negative. There is no certainty.

C) $\underset{(-)}{(a}+\underset{(+)}{c)}(a-c) \Rightarrow \underset{(-)}{(a}+\underset{(+)}{c)}(-) \Rightarrow$ There is no certainty.

D) $(a-c).\underset{(+)}{c} \Rightarrow (-).(+) \Rightarrow (-)$

E) $(\underset{(-)}{a}+\underset{(+)}{c}).\underset{(+)}{b} \Rightarrow (\underset{(-)}{a}+\underset{(+)}{c})(+) \Rightarrow$ There is no certainty.

Correct Answer: A

4)

Given that,

$a+b>0$

$b-c<0$

which of the following is definitely true for a, b, and c?

A) If a is a positive number, then b is negative.
B) If a is a negative number, then c is positive.
C) Both a and b are definitely positive numbers.
D) c is definitely a positive number.
E) If b is a negative number, then c is positive.

Solution:

Looking at the inequalities

$a+b>0$

$b-c<0$

let's try to interpret the question.

$a+b>0 \Rightarrow$ we cannot directly determine the signs of a and b.

$\Rightarrow$ If a is positive, then b can be negative or positive.

$\Rightarrow$ If a is negative, then b must be positive.

$b-c<0 \Rightarrow b<c \Rightarrow$ we cannot directly determine the signs of b and c.

$\Rightarrow$ If b is positive, then c must be positive.

$\Rightarrow$ If b is negative, then c can be negative or positive.

Therefore, the only statement that can be definitively concluded is:

If a is negative, then b is positive, and c is also positive $(-,+,+)$.

Correct answer: B

5)

Given that,

$a<0<b<c$

Which of the following is definitely negative?

A) $\frac{a+b}{b-c}$ B) $\frac{a+c}{a-c}$ C) $\frac{a+b}{c}$ D) $\frac{a-b}{b+c}$ E) $\frac{b-c}{a}$

Solution:

$a<0<b<c \Rightarrow \underset{(-)}{a}<0<\underset{(+)}{b}<\underset{(+)}{c}$

The sign rule in division is the same as in multiplication. Division of numbers with the same sign is positive; division of numbers with opposite sign is negative.

A) $\frac{a+b}{b-c} \Rightarrow \frac{\text{Not certain}}{(-)} \Rightarrow$ There is no certainty.

B) $\frac{a+c}{a-c} \Rightarrow \frac{\text{Not certain}}{(-)} \Rightarrow$ There is no certainty.

C) $\frac{a+b}{c} \Rightarrow \frac{\text{Not certain}}{(+)} \Rightarrow$ There is no certainty.

D) $\frac{a-b}{b+c} \Rightarrow \frac{(-)}{(+)} \Rightarrow (-)$

E) $\frac{b-c}{a} \Rightarrow \frac{(-)}{(-)} \Rightarrow (+)$

Correct Answer: D